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Troldborg, Niels; Gaunaa, Mac; Guntur, Srinivas

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Modelling the influence of yaw using a simple vortex rotor model

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Niels Troldborg, Mac Gaunaa, Srinivas Guntur
Department of Wind Energy, DTU Wind Energy, Frederiksborgvej 399, P.O. Box 49, 4000 Roskilde

Abstract

A simple analytical rotor model based on vortex theory is presented and used to investigate the main mechanisms for wind turbine rotors operating at yaw misalignment. The overall findings of the model is verified by comparing with an existing model as well as with results obtained using the actuator disc technique combined with full Navier-Stokes computations.

Objectives

The objective of the present work is to develop a simple model for including the effect of yaw misalignment on induced velocities and power production of a wind turbine and to verify it by comparing it to the model by Glauert [1] as well as to Navier-Stokes actuator disc computations.

Methods

A simple vortex model for yawed rotors:

The proposed vortex model is a generalization of the model originally developed by Øye [2]. The basic assumptions and governing equations behind the model is outlined below.

Basic assumptions:

- Potential flow (i.e. incompressible, irrotational and inviscid)
- Steady and uniform inflow
- Rotational effects neglected (infinite rotational speed)
- Wake expansion neglected
- Transport velocity of vorticity sheet is (see Figure 1): $V_{sheet}=½(V_1+V_2)$

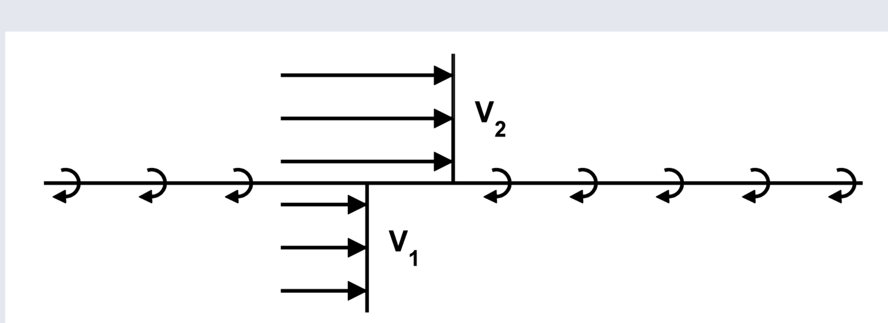

Figure 1: Sketch of vorticity sheet of strength $\gamma=V_2-V_1$

Governing equations:

The velocity field is everywhere governed by Biot Savart's law while the local forces per unit span on the blades are determined using Joukowski's relation:

$$dF = \rho V_{rel} \times d\Gamma$$

where dΓ is the local bound circulation, which is aligned with the blade, V_{rel} is the local velocity relative to the blade and ρ denotes density. Using these relations together with the above assumptions the following can be shown:

$$C_{T,loc} = \frac{F_n}{\frac{1}{2}\rho V_\infty^2} = \frac{\Gamma\Omega}{\pi V_\infty^2}$$

$$C_{P,loc} = C_{T,loc}\left(\cos\psi + \frac{W_n(r,\theta)}{V_\infty}\right)$$

where F_n signifies the local force per unit area in the normal direction and V_∞ denotes the free stream velocity. Γ signifies the total bound circulation on the rotor, ψ is the yaw angle and r and θ is the radial and tangential position where the forces are evaluated. W_n denotes the induced velocity in the normal direction at the rotor disc.

Navier Stokes actuator disc model:

The used actuator disc model combines the Navier-Stokes flow solver EllipSys3D with an actuator disc technique where body forces are distributed on a disc representing a wind turbine rotor [3]. The applied disc forces are distributed smoothly on several mesh point to avoid singular behavior. The steady state flow field is solved in a Cartesian computational domain with $18.9{\cdot}10^6$ number of grid points.

Results

Figure 2 compares actuator disc (AD) and vortex model (VM) predictions of $C_{P,loc}$ *and* $a_n = \cos(\psi) - W_n(r, \theta)/V_\infty$, respectively for $C_T = 0.64$ *and* $\psi = 30°$.

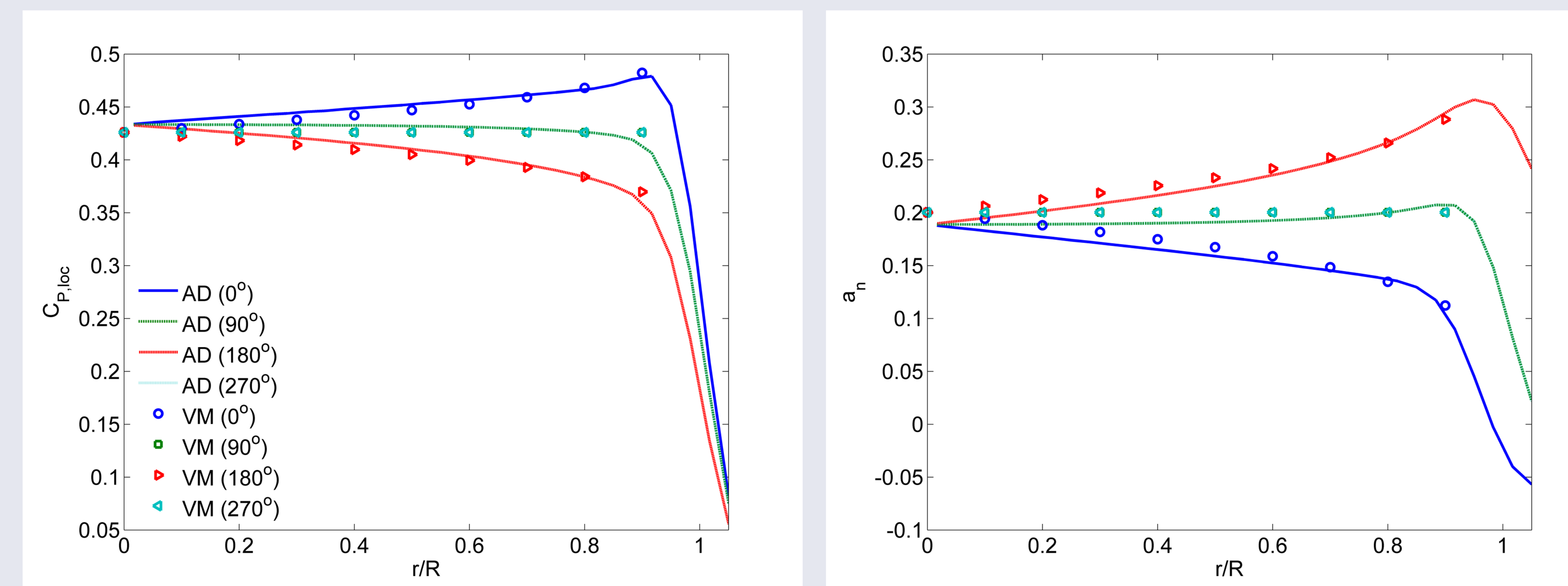

Figure 2: Local power coefficient (left) and normal induction in the rotor plane (right) predicted using the simple vortex model and actuator disc model, respectively. The thrust coefficient and yaw angle is $C_T = 0.64$ and $\psi = 30°$.

Figure 3 shows a comparison of the predicted inductions in the radial and tangential directions in the same case as in Figure 2.

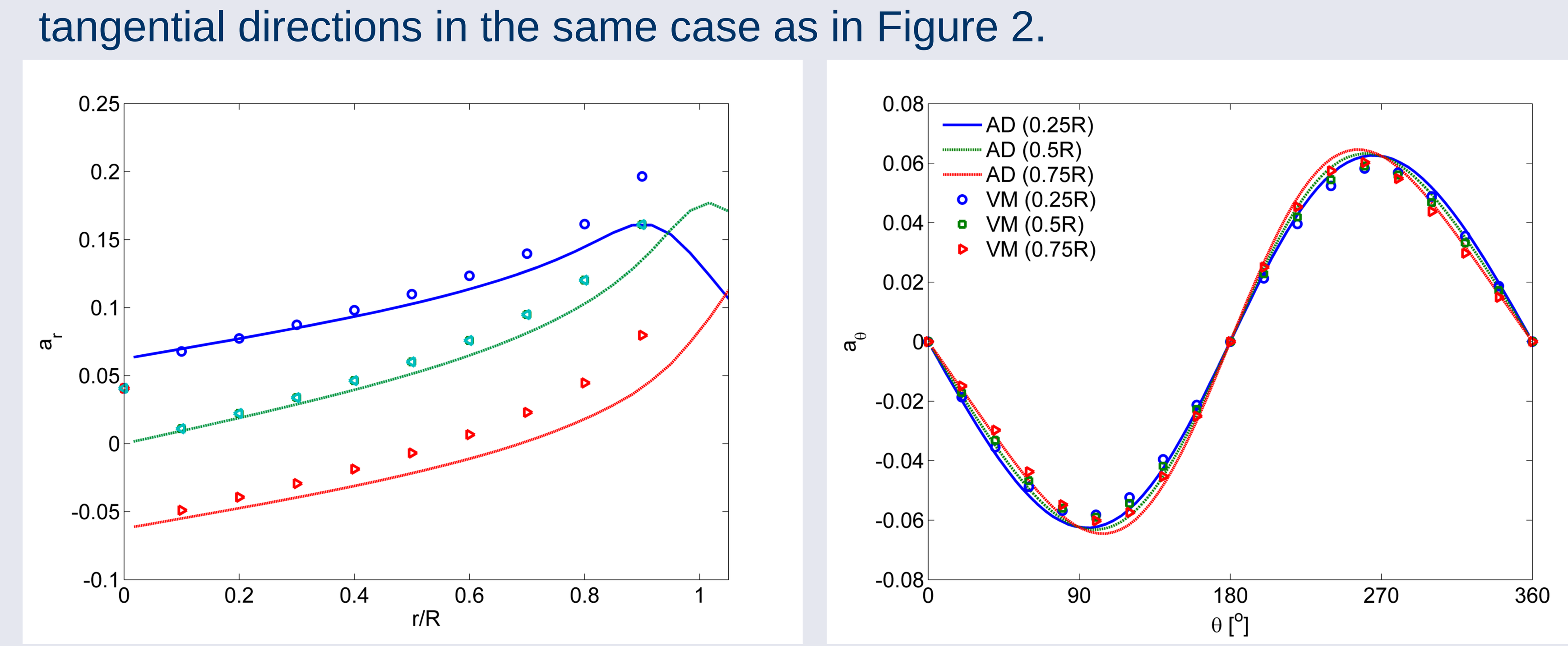

Figure 3: Radial (left) and tangential (right) induction in the rotor plane predicted using the simple vortex model and actuator disc model, respectively. The thrust coefficient and yaw angle is $C_T = 0.64$ and $\psi = 30°$. The color and symbol code in the left plot is as in Figure 2.

Figure 4 shows the global power coefficient as a function of yaw angle predicted by the vortex model and Glauert's model, respectively in comparison with actuator disc results.

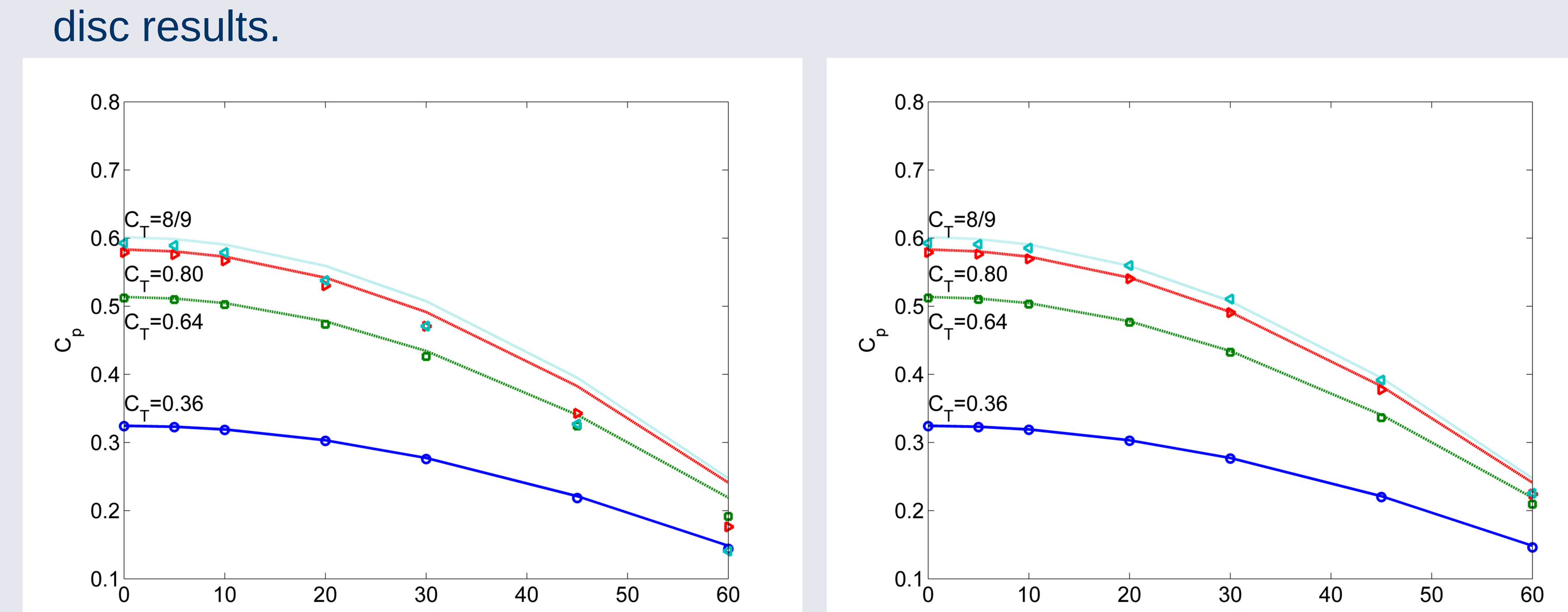

Figure 4: Power coefficient as a function of yaw angle for various thrust coefficients. Full lines are AD predictions and symbols are predictions by the vortex model (left) and Glauert's model (right).

Conclusions

The proposed vortex model is shown to give predictions of all induction components and power performance, which are in close agreement with actuator disc simulations at low to medium loads and over a wide range of yaw angles. However, for higher loads and yaw angles the vortex model is less accurate. In these cases the model is outperformed by the model of Glauert, which is shown to be accurate for all load conditions in terms of integral values. However, unlike the proposed vortex model, the model of Glauert does not provide the inductions in the radial and tangential directions and cannot predict the nonlinear behavior of the induced velocity on the outer part of the disc.

References

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3. Mikkelsen, R. Actuator Disc Methods Applied to Wind Turbines, MEK-FM-PHD 2003-02, Technical University of Denmark, 2003.